THE LYAPUNOV FUNCTIONS IN ESTIMATES OF A DIMENSION OF DYNAMICAL SYSTEM ATTRACTORS

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The present work was reported in the Rokhlin memorial conference in August 1999. It incorporates a review of the results, concerning estimations of attractor dimensions by the Lyapunov functions.

This line of investigation made possible to obtain the highly simple formulas for computing the Lyapunov dimension of the Henon and Lorenz attractors

Consider the Henon mapping $F : \mathbb{R}^2 \to \mathbb{R}^2$

$$\begin{aligned} &z \to a + by - z^2, \\ &y \to z, \end{aligned}$$

where $a > 0$, $b \in (0, 1)$. We shall consider here the Lyapunov dimension $\dim_L K$ of an invariant set K: $FK = K$, containing the following stationary points

$$z_\pm = \frac{1}{2}\left[b - 1 \pm \sqrt{(b-1)^2 + 4a}\right].$$

Theorem 1. *For the mapping F the equality*

$$\dim_L K = 1 + \frac{1}{1 - \ln b / \ln \alpha(z_-)},$$

where $\alpha(z_-) = \sqrt{z_-^2 + b} - z_-$, is valid.

We now consider the Lorenz system

$$\begin{aligned} \dot{x} &= -\sigma x + \sigma y, \\ \dot{y} &= rx - y - xz, \\ \dot{z} &= -bz + xy, \end{aligned} \tag{1}$$

where r, b, σ are positive numbers. Suppose that

$$\begin{aligned} &b \geq 2, \qquad \sigma + 1 \geq b, \\ &r\sigma^2(4 - b) + 2\sigma(b-1)(2\sigma - 3b) > b(b-1)^2 \end{aligned}$$

and denote by K an invariant set of system (1), containing the point $x = y = z = 0$.

Theorem 2. *The following equality holds*

$$\dim_L K = 3 - \frac{2(\sigma + b + 1)}{\sigma + 1 + \sqrt{(\sigma - 1)^2 + 4r\sigma}}.$$