THE BROCKETT STABILIZATION PROBLEM

Gennadij A. Leonov

Dept. of Mathematics and Mechanics
St.Petersburg State University
2, Bibliotechnaya pl., Peterhof
St.Petersburg 198504, Russia
leonov@math.spbu.ru

Abstract

R. Brockett has formulated in book "Open problems in mathematical systems and control theory" (Springer 1999) the following problem.

Given a triple constant matrices A, B, C under what circumstances does there exist a time-dependent matrix $K(t)$ such that the system

$$\frac{dx}{dt} = Ax + BK(t)Cx, \quad x \in \mathbb{R}^n \tag{1}$$

is asymptotically stable.

In a number of cases the answer to this question can be found by means of the following results.

Theorem 1. *Let there exist matrices K_1 and K_2 satisfying the following conditions:*

1) a matrix BK_1C has $n-1$ eigenvalues with negative real parts and $\det BK_1C = 0$,

2) for a vector $u \neq 0$, satisfying an equality $BK_1Cu = 0$, and for a certain number λ, the vector-function

$$\exp\left[(A + BK_2C + \lambda I)t\right]u$$

is periodic.

Then there exists a periodical matrix $K(t)$ such that system (1) *is asymptotically stable.*

Theorem 2. *Let $n = 2$ and there exist matrices K_1, K_2 such that the following conditions hold*

1) $\det BK_1C = 0$, $\operatorname{Tr} BK_1C \neq 0$,

2) a matrix $A + BK_2C$ has complex eigenvalues.

Then there exists a periodic matrix $K(t)$ such that system (1) *is asymptotically stable.*

Corollary. Let $n = 2$, B is a column vector, C is a row vector, $K(t)$ is a function, the transfer function has the form

$$W(p) = C(A - pI)^{-1}B = \frac{p + \gamma}{p^2 + \alpha p + \beta},$$

where $\gamma \leq 0$, $\alpha\gamma > \beta$.

If

$$\gamma^2 - \alpha\gamma + \beta > 0$$

then there exists a periodic function $K(t)$ such that system (1) is asymptotically stable.

If

$$\gamma^2 - \alpha\gamma + \beta < 0$$

then there do not exist functions $K(t)$, for which system (1) is asymptotically stable.

Theorem 3. *Let $n = 3$, B is a column vector, C is a row vector, $K(t)$ is function, $CB \neq 0$ and there exist a numbers K_1, K_2 satisfying the following conditions:*

1) *a matrix $A + K_1BC$ has two complex eigenvalues and one negative eigenvalue,*
2) *function*

$$C \exp[(A + K_2BC)t]B$$

has at least one zero on interval $(-\infty, 0)$.

Then there exists a periodic function $K(t)$ such that system (1) is asymptotically stable.